The use of logsums in welfare estimation: application in PRISM

Serbjeet Kohli
ITS, University of Leeds and Mott MacDonald
Andrew Daly
ITS, University of Leeds and RAND Europe

1. INTRODUCTION

Welfare estimation lies at the heart of most appraisal processes in transport planning. The use of logit models based on random utility maximisation (RUM) opens up a new dimension in the welfare estimation techniques with the use of the "logsum" as an evaluation measure.

In this paper we discuss the problems arising in applications of the logsum technique to evaluate consumer surplus under different transport planning scenarios. The model used for the applications is PRISM, the new strategic transport model of the West Midlands in the U.K. (see van Vuren *et al*., 2004).

In the following section of the paper, we set out briefly the economic basis of consumer surplus and explain how modelling travel demand through random utility maximisation, and in particular through the use of GEV models (McFadden, 1978), leads to the logsum which meets the need for a consumer surplus measure. A very brief review is given of possible extensions to deal with more general models.

Section 3 gives a brief outline of the PRISM model and its sub-model representing commuter travel, which has been used for the tests made; details are also given of the process used to calculate consumer surplus. In practical models, practical problems arise and in this case the issue arose of the conversion of the logsum to a monetary scale, given the non-linear form in which mode costs appear in the model.

The following section presents a test which studies the impact of two network interventions which improve the transport infrastructure in the West Midlands.

Section 5 presents some of the key benefits and disbenefits of the logsum measure both from theoretical and practical perspective.

We conclude that apart from the issues of cardinality assumptions and non-linear formulation of variables the logsum method proves to be theoretically accurate and practically useful.

2.THE ECONOMIC BASIS OF THE LOGSUM MEASURE

The use of discrete choice models in the transport field has seen a significant increase in recent years. The economic theory of Random Utility Maximisation (RUM) underpinning these disaggregate models has been used extensively in transport demand modelling applications in various forms in the past. These applications range from simple mode choice to more complex mode, destination and time-of-day choice. However the extension of modelling applications into economic appraisal using RUM directly has been quite sparse.

The economic theory of RUM is very clear and is based on the concept of a rational utility-maximising economic agent. Probabilistic choice theory quite elegantly extends the utility-maximising notion under discrete choice situations to make the theory applicable. One of the primary reasons for the use of discrete choice models in the travel demand modelling field is due to the core nature of the theory which deals with 'individual choice'. The concept of deriving social surplus using the RUM with probabilistic discrete choice models has been present in the theory for a quite some time but has found few applications. Partly responsible for this neglect has been the complexity of the disaggregate models found in the applications relative to the conventional aggregate models. A few applications have been made recently (see de Jong *et al*., 2005, and the literature reviewed there), but this paper presents the first large-scale application in the UK of which we are aware.

With a few basic assumptions on the structure of utility functions, RUM can be extended to a computationally tractable form to provide the total consumer surplus as the '*logsum*' in a discrete choice model. The strength of the logsum as a measure of consumer surplus lies in the variety of attributes of the alternatives that it can encapsulate within a single term. Depending upon the segmentation structure of the choice model the logsum term can also generate consumer surplus estimates for different market segments simultaneously.

2.1 Consumer surplus

To determine the worth of the implementation of a particular project or policy we need to compare what consumers are willing to pay to gain the benefits from the project against the actual costs of the project. The consumers' willingness to pay is considered to be the satisfaction that the consumers gain by the implementation of the project and is known as the gain in 'consumer surplus'. The concept of consumer surplus in welfare economics was originally formulated by French economist Dupuit (1844), coincidentally in discussing the value of public expenditure in a transport context. More formally the "willingness to pay" interpretation of the consumer surplus measure was demonstrated by Marshall (1920). Marshall treated consumer surplus as the excess price which a consumer will be willing to pay, over and above its market price, rather than go without the good as the measure of the consumer surplus. In the case of change in prices of a good from P_1 to P_2 due to implementation of a project the gain in consumer surplus is given by the shaded area in Figure 1.

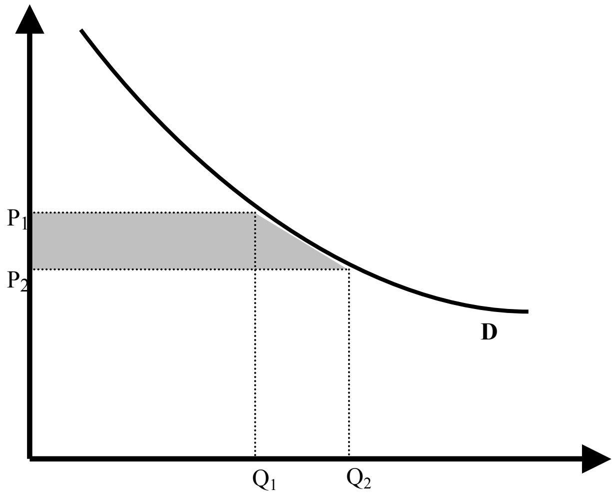

Figure 1: Consumer Surplus

For a linear approximation of a downward sloping demand curve, as shown above, the value of the consumer surplus can be estimated as the area under the demand curve using the well-known Rule of Half:

$$CS = \frac{1}{2}(Q_1 + Q_2) * (P_1 - P_2)$$

where: Q_1 and Q_2 are quantities of the good demanded at prices P_1 and P_2. Nearly all practical applications in the transport field use the Rule of Half to evaluate change in consumer surplus.

These Marshallian or 'uncompensated' demand curves subsume two distinct effects on consumer's choice: substitution effect (from other goods) and income effect (from increased/decreased purchasing power). As a result, if there are large income effects due to a price (or quality) change, the area under Marshallian demand curves does not accurately represent the compensation that the consumers are willing to accept (or pay) for a price (or quality) change which would leave them at their original levels of satisfaction.

In contrast, a compensated demand curve represents "a pure substitution effect, which is always to increase demand of a good whose price has fallen" (Pearce and Nash, 1981 p93). The uncompensated demand curve represents both substitution and income effects. If the income effects of a price change for a particular good are small, e.g. if the expenditure on the good is a small proportion of the total income, then the compensating variation (CV) can be considered to be equivalent to the Marshallian consumer surplus (CS). Although CV, originally developed by Hicks (1956), has been considered to be the more appropriate measure for evaluating welfare changes, CS tends to be the most widely used measure in practice. Marshallian demand curves are easier to observe and estimate and also for several goods income effects are relatively small, hence CS suffices as a welfare evaluation measure. Under these assumptions the rule of a half formulation for the calculation of CS, with a linearity assumption for the uncompensated demand curve, has generally been considered to be acceptable.

2.2 Random utility maximisation (RUM)

Random Utility Maximisation (RUM) is derived from individual choice theory under discrete choice situations. Marschak (1960) and Block and Marschak (1960) formulated the first RUM based models using a probabilistic choice theory of individual choice. Consumers are assumed to have consistent and transitive preferences over the alternatives available in their choice set. The attractiveness of each alternative can then be translated into a real valued monotonic function called the 'utility function'.

It is important to note that the category of utility being discussed here is ordinal utility which can be subjected to any order-preserving transformation. Under continuous choice situations the utility maximisation assumptions can be shown to result in classical demand curves. However with discrete choice situations, i.e. where the alternatives are mutually exclusive, direct integration is not possible and therefore the utility functions are used directly to model consumer behaviour. Within the discrete choice framework the choice sets consist of alternatives that are mutually exclusive, exhaustive and finite.

A key issue in the definition of utility functions is that an observer does not know the exact value of these utilities, as there are elements in the actual utilities that cannot be observed. For this reason, the utility function is formulated as:

$$U_{in} = V(x_{in}, s_n) + \varepsilon(x_{in}, s_n) = V_{in} + \varepsilon_{in}$$

where: U_{in}: Utility of alternative *i* for an individual (decision maker) *n*;
x_{in}: Attributes of alternative *i*;
s_n: Socio-economic characteristics of individual *n*;
V_{in}: is the observed (systematic) element of utility; and
ε_{in}: is the unobserved element of utility.

The observer can consider the unobserved element ε of the utility to be randomly distributed over the population. Manski (1973) gives four explicit sources of the randomness:

1. unobserved attributes of the alternatives and the individuals;
2. unobserved taste variation among the individuals making decisions;
3. unobserved errors and imperfect information of the attributes of the alternatives and the individuals; and
4. unobserved random errors in the relationship between the proxy variables used instead of actual attributes and the attributes itself.

With randomness in the utilities the selection of the highest utility alternative can now be expressed only in a probabilistic form, by assuming some distribution of the error term.

However, the specification of this distribution of the random terms invalidates the ordinal utility assumption and converts the utility specification to cardinal utility. Batley (2006) discusses the cardinal vs. ordinal utility structure used in applied RUM models and concludes that most of the implementations of RUM carry cardinal utility characteristics. The debate on reconciliation of these cardinality assumptions of utilities, taken to make RUM applicable, with classical welfare economics based ordinal utilities is ongoing.

2.3 The logsum as a measure of consumer surplus

The choice probability function under a RUM framework can be considered as the expected uncompensated demand curve of a particular alternative (Small and Rosen, 1981). Consumer surplus can be defined as the maximum utility, in monetary terms, that an individual receives by choosing the alternative in a choice situation. Therefore consumer surplus will be equivalent to the indirect utility of the alternative that is chosen by the individual to maximise the individual's own utility. Mathematically:

$$CS_n = \frac{1}{\alpha_n}\left(\max_i\left(U_{in}\forall i\right)\right)$$

where: CS_n: consumer surplus of individual n;

α_n: marginal utility of income of individual n i.e. dU_n/dY_n where Y_n is the income of individual n and U_n is the overall utility; and

U_{in}: utility of individual n when alternative i is chosen.

As the value of the utility is not observed only the expected consumer surplus ($E(CS_n)$) can be estimated by:

$$E(CS_n) = \frac{1}{\alpha_n}E\left(\max_i\left(U_{in}\forall i\right)\right)$$

Under the condition of a translationally invariant distribution of the random elements of utility (ε_{in}) Williams (1977) and Ben-Akiva and Lerman (1979) have shown that:

$$\frac{dE\left(\max_i\left(U_{in}\right)\right)}{dV_{in}} = P_{in}$$

where: P_{in}: Probability of alternative i being chosen by individual n; and

V_{in}: is the observed (systematic) element of utility of alternative i.

For identically and independently distributed Gumbel disturbances, which is the logit model formulation, Williams (1977) and Small and Rosen (1981) show that:

$$\frac{dE\left(\max_i\left(U_{in}\right)\right)}{dV_{in}} = P_{in} = \frac{e^{V_{in}}}{\sum_i e^{V_{in}}}$$

$$E\left(\max_i\left(U_{in}\right)\right) = \int \frac{e^{V_{in}}}{\sum_i e^{V_{in}}} = \ln\sum_i e^{V_{in}} + C$$

$$E(CS_n) = \frac{1}{\alpha_n}E\left(\max_i\left(U_{in}\right)\right) = \frac{1}{\alpha_n}\ln\sum_i e^{V_{in}} + C$$

where C is an unknown constant representing the fact that absolute level of utility cannot be measured. The expression ln $\sum e^{Vin}$ effectively gives the consumer surplus for an individual in a logit model formulation – hence the name "logsum". $E(CS_n)$ is the average consumer surplus of the segment of population which is represented by individual *n*, i.e. the representative utilities (V_{in}) of the population segment are same.

The common approach for calculating the total consumer surplus is by taking a weighted sum of the average consumer surpluses where the weights are the numbers of people in each segment which have the same representative utilities (Train, 2003). Here the assumption is of equal weights given to each segment of society and hence a pure linear addition of consumer surpluses not only implies cardinality of utilities, but also leads into a controversial value judgement on the distributive impacts of policy. One benefit of obtaining surpluses for different segments, by income or other variables, is the capability to inform the decision maker regarding the distributive impacts of the policy.

Apart from these issues, in simple terms the integral of logit choice probability is the logsum. Since the choice probability becomes the demand curve in discrete choice situations, the logsum becomes the consumer surplus (area under the demand curve).

McFadden (1978, 1981) extended the RUM based models by formulating the Generalised Extreme Value (GEV) family of models which allow for correlation of the unobserved elements of utilities of the alternatives. McFadden's GEV theorem states that:

> **If** G is a non-negative and linear homogenous of-degree μ (μ>0) function of $Y_i \geq 0$ (where $Y_i = \exp V_i$ with V_i being the representative utility of alternative i) i.e.
>
> $$G(\alpha Y_1, \alpha Y_2, \ldots\ldots, \alpha Y_n) = \alpha^{\mu} G(Y_1, Y_2, \ldots\ldots, Y_n);$$
>
> **such that** $\lim_{Y_i \to \infty} G = \infty \ \forall\ i$; **and** the mixed partial derivatives of G exist and are continuous with non-positive even and non-negative odd mixed partial derivatives;
>
> **then** the probability of alternative i being chosen by an individual is given by:
>
> $$P_i = \frac{Y_i \left(\dfrac{\partial G}{\partial Y_i} \right)}{\mu G}:$$
>
> **and** the average utility of the chosen alternative, which is equivalent to consumer surplus, is given by $\dfrac{\log G}{\mu}$.

A large number of formulations of G have been created to give different model forms. For example $G = \sum Y_i$ gives the simple multinomial logit model (MNL) and log G is the logsum measure as discussed above. For more complex G functions log G gives a generalised form of logsum for the family of GEV models.

A very common application of GEV models is in the form of the nested logit model where the alternatives are considered as ordered in a nested tree structure. The utilities of alternatives in different nests are considered to be iid but not within a nest. If *i* alternatives are partitioned in *m* non-overlapping subsets (nests) B_1, B_2,…, B_m then $G = \sum_{1}^{m}\left(\sum_{i \in B_m} Y_i\right)^{\lambda}$ gives a two-level nested logit GEV model with (1–λ) representing a measure of the correlation in the unobserved utilities of the alternatives in nest *m*. In this model the homogeneity parameter $\mu = \lambda$.

Further extensions to the nested logit model can be achieved by the use of a multi-level nesting structure. For example, a three level nest structure in a mode choice situation could be applied by partitioning the alternatives into highway (car-driver, car-passenger) and public transport (train, bus) nests and then further partitioning the public transport alternatives into access mode alternatives like car-driver access to train, car-passenger access to train etc.. In these models the homogeneity parameter μ would be given by the product of all the structural parameters. Note that this property derives from the way in which the model is normalised and different valid specifications are also possible (see Daly, 2001).

The key result for each of the GEV based models is that a logsum formulation exists which can be used for consumer surplus calculation after correcting by the homogeneity parameter.

2.4 Further theoretical developments

The logsum formulation as a welfare benefit measure (CV) has also been derived by Small and Rosen (1981) from first principles for price and quality changes. In cases where income effects are considered to be an important aspect of appraisal of a policy or an intervention (Jara-Diaz and Videla, 1990) or income is considered to enter the utility functions in a non-linear form (Herriges and Kling, 1999) more complex formulations and procedures are required to derive consumer surplus measures from discrete choice models.

Most of the early research in RUM from the 1970's to the 1990's relied upon the linearity assumption of income in utility functions. The reason for imposition of this assumption can partly be explained by the complexities involved in estimating non-linear models and the difficulty in computing welfare benefits with these models. Another possible reason for using this assumption could be that the development and utilisation of most RUM models have been in the developed part of the world and the policies and interventions appraised using these models are generally are not considered to change users costs relative to their income. The primary developments under these assumptions in RUM models during this period were on generalisation of distribution of the random errors in utility definition to develop more complex models.

More recently during late 1990's and 2000 onwards further theoretical developments have taken place where the linearity of income assumption has been relaxed and methodologies developed to estimate welfare benefits from RUM models with non-linear income effects. One of the key contributions in this field has been by McFadden (1995, 1997, 1999). McFadden demonstrates the difficulty in estimating exact willingness to pay as the income reduction that keeps the consumer's expected utility constant in before and after conditions when the utility function is non-linear in income. The standard logsum formulations discussed earlier are no longer applicable to evaluate consumer surplus in this case.

Herriges and Kling (1999) have further investigated approaches available for calculating welfare benefits from RUM models with non-linear income effects. Karlstrom (2000) presents an alternative approach with direct calculation for GEV models. Further theoretical research in this area is ongoing, e.g. by Daly (2004) and Dagsvik and Karlstrom (2005).

3. PRISM: THE MODEL USED

PRISM (**P**olicy **R**esponsive **I**ntegrated **S**trategic **M**odel) West Midlands was developed between 2002 and 2004 by Mott MacDonald and RAND Europe to support the policies of the West Midlands local authorities, CENTRO (the public transport authority) and the Highways Agency (see van Vuren, 2005). It is a disaggregate multi-modal transport model with a detailed network representation.

PRISM's primary study area is the former West Midlands County, also referred to as the West Midlands metropolitan area, which comprises seven metropolitan districts. The highway and public transport networks of PRISM extend to cover the entire UK but the level of detail in network representation is much higher in the areas within and in the immediate vicinity of the West Midlands metropolitan area.

The version of PRISM used in this study is the version currently being used for applications. Further developments of the model to incorporate income segmentation, car-passenger cost sharing etc. are currently ongoing and these will be particularly important for investigating the distributional effects of policy in conjunction with the use of logsums.

3.1 Structure of PRISM

PRISM can be viewed as a system of models, with a number of sub-models interacting with each other, common to several strategic models. A simplified model structure is illustrated in Figure 2.

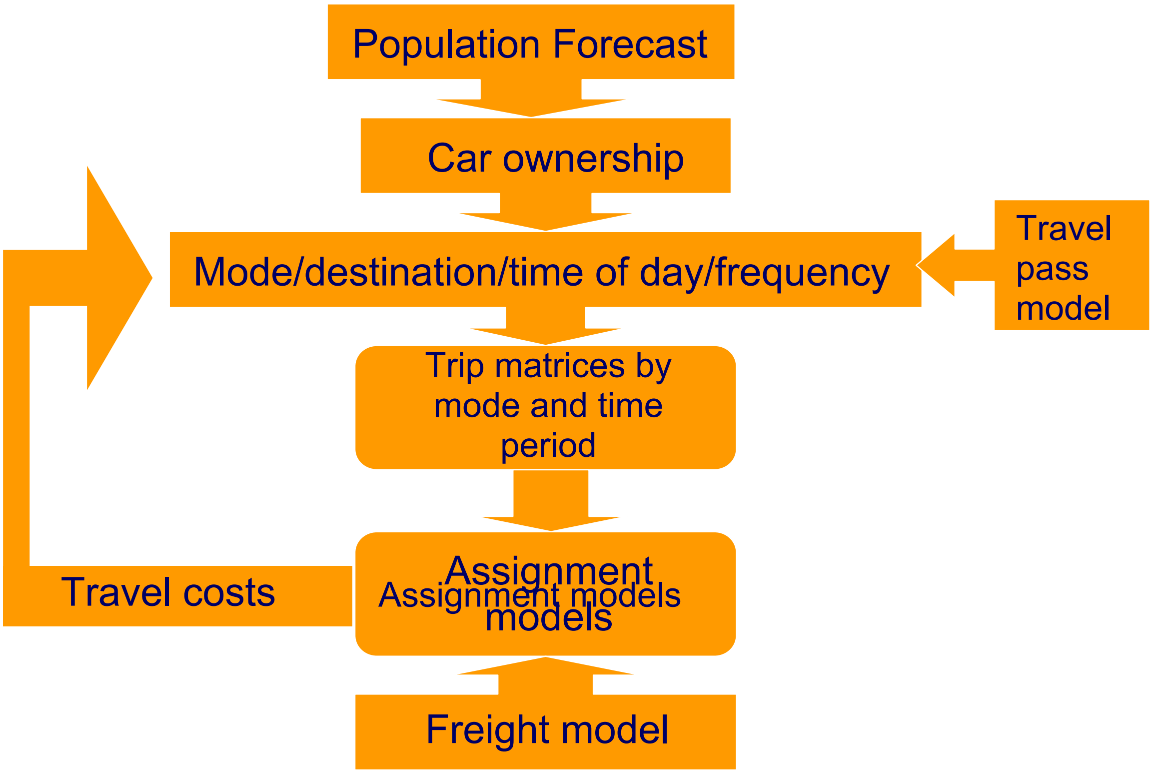

Figure 2: Structure of the model
adapted from: PRISM Freight Model Presentation (2006)

The travel demand model of PRISM consists of the following key components (Daly *et al*., 2004):

- ***Population and Car Ownership Forecast Model***: which predicts the detailed composition of the future West Midlands population using prototypical sample enumeration techniques (Daly, 1998) and the growth in car ownership in the West Midlands using the model developed for UK National Transport car ownership forecasts (Whelan, 2001).
- ***Travel Demand Forecasting Models***: a series of sub-models for different travel purposes modelling travel responses of the predicted population such as public transport (PT) pass ownership, mode and destination choice, time period choice, tour frequency and PT access mode and station choice. The travel demand model system of PRISM consists of:
    - 6 home-based tour models for commute, other, shopping and education (3 education models) travel;
    - 3 trip models for non-home based employer business, non-home based other travel and home-based employer business trips; and
    - an Airport trip origin and mode choice model.
- ***Final Processing Model***: converts the travel demand model outputs to trip matrices for different modes and purposes and pivots with respect to base matrices (Daly *et al*., 2005) to give forecast matrices to be assigned on the network model.

Each of the home-based tour models in PRISM includes frequency, mode, destination and time-of-day choice responses except for the education travel models that do not have the time-of-day choice component in their structure. The unit of travel for all home-based travel, except business travel, is a home-based tour, defined as a series of linked journeys starting and finishing at the traveller's home.

Amongst the 9 different travel demand models in PRISM the home-based commute tour model was chosen for this study for calculation of consumer surplus using logsums. The main reasons for use of the commute model were the following.

- It covers all the primary demand responses such as PT pass ownership, mode, destination, time-of-day, PT access mode and station choice and frequency.
- It has a very detailed segmentation of the population by car availability, pass ownership etc. which is useful in assessment of the impact of a policy or a scheme on different users.
- A large number of policies and schemes are focused on relieving the peak period problems such as congestion, hence primarily benefiting the commuters. Therefore analysis of the commuter benefits would be a good indication of the overall benefit of the policy or the scheme being tested.
- The resources available for testing and analysing the logsum method were limited hence the study concentrated on a single model only.

The commuter model is discussed in further detail in the section below.

3.2 PRISM commute model

The home-based commute tour model is a nested logit model. Eight modes are represented:

- car driver, for which a time period choice model is also applied;
- car passenger;
- train, which includes metro and bus as possible access modes, for which also access mode choice and station choice for car drivers and car passengers are modelled;
- metro, which may include bus as an access mode, for which access choices are modelled as for train;
- bus, for which the only access considered is by walking;
- taxi;
- cycle; and
- walk.

In addition to modelling choices among modes, time periods and access choices, the model predicts choice of destination and travel frequency. All of these choices are fully linked in the nested logit structure.

The models were estimated from a combination of conventional home interview, Stated Preference and station intercept surveys. The estimation procedure was somewhat complicated, because of the combination of data types and the need to estimate the structural parameters. However, in the final specification used for model application the homogeneity parameter μ took the value 1, so that the logsums do not need to be scaled to recover the scale of the alternative utilities.

To reflect different behavioural characteristics of individuals which affect their travel behaviour it is essential to segment the population by various socio-economic characteristics. The dimensions of segmentation in the commute model are: car availability, working status, gender, pass ownership and occupation. The individual segments under these dimensions are as shown in Table 1.

Table 1: Segmentation in the Commute model
(I) Mode-Destination segmentation:

Car availability (6 segments)	**Working Status** (2 segments)	**Gender** (2 segments)	**Pass ownership** (5 segments)
No cars in household	Full time worker	Male	Centrocard
No licence, one-plus cars	Part time worker	Female	Bus-only pass
Licence, one car, free car use			Rail-only pass
Licence, one car, car competition			Other pass types (railcards for train, blind/disabled for metro and bus)
Licence, two-plus cars, free car use			
Licence, two-plus cars, car competition			No pass

(II) Frequency segmentation: mode-destination segments, plus:

Occupation (3 segments)
Manager/professional, skilled manual/foreman/supervisor, other manual
Other clerical non-manual
Other occupation types

The population forecasting model predicts the number of people in each of the 72 segments (car availability working status, gender and occupation, i.e. 6*2*2*3) for each of the home zones in the modelled area. The pass ownership model is used to calculate the probabilities of owning different types of PT passes for each mode-destination segment as the pass owning determines the PT cost of travel in the model.

The commute tour frequency model is applied at each individual segment level for each home zone. The frequency model has an accessibility component so that an improvement in accessibility from a particular zone results in increased number of tours. However the difference is minuscule (0.02%) in most cases as the accessibility coefficient in the frequency model is quite small.

In the model estimation stage a large number of utility formulations were tested using the commuter tour data available from the data sources. This process led to a rich range of variables included in the utility specification of the model. One important variable to note here is the cost variable which has been included in the model in logarithmic form. The primary reason for using the log cost formulation is the better model fit obtained by this specification

(Fox, 2004). This formulation has important implications for the logsum method used to calculate consumer surplus which are discussed in more detail in the section below.

3.4 Calculation of the consumer surplus

The consumer surplus for each particular segment *s* in each home zone *o* is calculated by multiplying the logsum difference between the “Do Something” scenario (superscript 1) and the “Do Minimum” scenario (superscript 0) by the number of tours made by people in that segment:

$$CS^u_{os} = \tau^1_{os}\left(\ln\sum_n \exp(V^1_{ons}) - \ln\sum_n \exp(V^0_{ons})\right)$$

where: CS^u_{os} : consumer surplus (in the units of *V*);

τ^1_{os} : number of tours; and

V^1_{ons}, V^0_{ons}: representative utility of alternative *n* in the relevant scenarios.

It should be noted that the logsum is calculated over the mode, destination and time period choices, i.e. excluding the frequency model. This is done because the logsum from the frequency model would not be very responsive to travel time changes, as the coefficient in that model is very small. In consequence the formula above is calculated over tours, not over persons. The frequency model predicts tours in the two scenarios that depend slightly on the accessibility measure in each case, thus complicating the consumer surplus calculation. Calculation for the consumer surplus using an average number of tours ((τ^1_{os} + τ^0_{os})/2) was carried out but the results were not found to be significantly different and “Do Something” tours (τ^1_{os}) have been used in the calculations.

As discussed above the commute model in PRISM has a log cost variable in its utility definition. The non-linearity of the cost variable causes a problem in calculating the marginal utility of income/money which is required for monetising the maximum utility difference to give the consumer surplus in monetary terms. Part of the utility function is:

$$U_{in} = \alpha_t Time + \alpha_c Log(Cost) + ...$$

So that the marginal utility of income is:

$$\frac{\partial U_{in}}{\partial Cost} = \alpha_c \frac{1}{Cost}$$

which complicates the monetisation of logsums as the cost varies over different alternatives. As an approximation de Jong et. al. (2005a) and Rohr (2006) suggest evaluation of the cost coefficient as:

$$\frac{\partial U_{in}}{\partial Cost} = \alpha_c E\left(\frac{1}{Cost}\right)$$

where E(1/Cost) is the expected value of the 1/Cost per tour for a alternatives which have a cost, averaging over modes and destinations. The assumption is thus that users of modes which do not have a cost would have the same marginal disutility of expenditure. To convert the consumer surplus from logsum units to money units the following approximation is used:

$$CS_{os}^{m} = \frac{CS_{os}^{u}}{\alpha_c E\left(\dfrac{1}{Cost}\right)} = \frac{\tau_{os}\left(\ln\sum_n \exp(V_{ns}^{1}) - \ln\sum_n \exp(V_{ns}^{0})\right)}{\alpha_c E\left(\dfrac{1}{Cost}\right)}$$

where now CS_{os}^{m} gives the consumer surplus of users in home zone *o* and segment *s* in money units. Here there is an implicit assumption that the base-year cost average is applicable for both future-year scenarios, but it seems necessary to keep the cost scale uniform to avoid perverse effects.

An alternative method to monetise the logsums based consumer surplus is also suggested by de Jong *et al*. (2005a). This involves converting the consumer surplus to time units by using the travel time coefficient and then applying a value of time (VOT) estimate to obtain benefits in monetary terms:

$$CS_{os}^{m} = \left(\frac{CS_{os}^{u}}{\alpha_t}\right) * VOT$$

The VOT estimate used can be taken either from the values implied by the model or from published values. This process avoids the difficulty caused by the linearisation procedure.

4. RESULTS OF THE TEST

The tests were made in the context forecasts for the year 2021 made by the PRISM application team. It should be stressed that the results obtained in these tests are purely illustrative, as only commuter travel has been used and a full pivoting procedure has not been carried out for the results presented here, although the runs are fully equilibrated. Further, the benefits are calculated for an average working day and have not been annualised or discounted. No costs of implementation or operation have been calculated.

The 2021 Reference Case, which was used as the “Do Minimum”, incorporated a substantial number of network improvements (WMRA, 2006). Additionally, a 9% growth in the number of households and 11% growth in employment across the West Midlands area are assumed, varying over the study area, with suitable assumptions relating to demographic developments, income and car ownership.

To calculate the change in consumer surplus for the commuters as a result of the implementation of “Do Something” scenario the methodology outlined in the previous section was used. All the results are presented for six car availability segments and five PT pass-ownership segments. One of the benefits of obtaining benefits by these segments is the ability to assess the impact on different segments of society. Also the impacts on public transport users can be inferred from the benefits calculated for different pass owning segments. A key benefit of this analysis as a policy instrument is the information that a decision maker can have regarding the distributional consequences of a scheme or a policy.

4.1 Motorway Box ATM Scheme

The West Midlands conurbation is flanked by motorways, including the M6, M5, M42, M40, M69 and M54 which form a motorway box. These motorways connect different parts of the region and also play an important role as national and international transport connections. Parts of the motorway network suffer from severe congestion problems especially during the peak periods. The UK Highways Agency has implemented an innovative scheme on the M42 J3a-J7 corridor known as Active Traffic Management (ATM) which improves the capacity of the motorway network by the combined use of technology and of hard-shoulder running under highly congested situations at a relatively lower cost than capacity expansion by adding lanes.

The Highways Agency intends to extend the ATM scheme across the rest of the motorway box network and this extension is what is tested here. The scheme is modelled on the basis that hard-shoulder operation would be required during the AM and the PM peak periods (3 hours each). This results in approximately 22% increase in capacity during the peak periods on the motorway sections where the scheme is implemented.

4.1.1 Scheme impacts

The demand model outputs of the model indicate a small overall shift from public transport modes (train, metro and bus) to car drivers and car passengers. There is also a time-of-day shift effect with the AM and PM periods for the “Do Something” scenario showing higher numbers of car driver trips than in “Do Minimum”.

The biggest impact is seen in the assignment results, with several motorway sections attracting significantly higher levels of traffic as a result of ATM implementation, and corresponding changes on competing and feeding routes. The ATM scheme has a mixed impact on the local road network with several of the inter-urban connecting routes in the conurbation seeing decreases in traffic volume and in other cases roads which serve as an access to the motorways having an increase in traffic volume.

4.1.2 Logsum benefits

The estimated benefits of the scheme using the logsum measures are shown in Table 2, separately for the various car availability and PT pass ownership segments (defined in Table 1).

Table 2:Logsum-based benefits of ATM scheme
(per average day)

Benefits in £'s		Car Availability Segments						
Cost coefficient (0.508)		**1**	**2**	**3**	**4**	**5**	**6**	**Total**
Pass Ownership Segments	**1**	2	12	14	14	41	8	**91**
	2	9	45	28	59	79	35	**254**
	3	3	25	11	13	32	8	**91**
	4	1	2	2	3	7	1	**16**
	5	138	583	1,581	1,837	4,418	951	**9,509**
	Total	**154**	**667**	**1,636**	**1,926**	**4,575**	**1,003**	**9,962**

It can be seen from Table 2 that the majority of the benefit accrues to those in pass ownership category 5 (no pass), which in fact covers 75% of commuter tours. This scheme is of little benefit to public transport users – only those using park-and-ride via motorways will benefit directly. Similarly, there is little benefit to those in car availability segments 1 and 2 (no car or no licence), although some of these people may be car passengers or even taxi users. The main beneficiaries are those in car availability segment 5 (2+ cars, free car use), which is as we would expect – this scheme would benefit heavy car users.

4.1.3 Rule-of-half calculations

Rule-of half calculations were also made for this simple scheme as a check on the logsum calculations and to illustrate the strengths and weaknesses of each approach.

The users benefiting most from the ATM scheme will be car-drivers. However some benefit will also accrue to car passengers and taxi users. Public transport and slow mode users will not be gaining any benefits except for the park-and-ride or kiss-and-ride trips which will be accounted for in the car-driver and car-passenger trip matrices for the home-station parts of their journeys. Therefore the ROH calculation was carried out for car-driver, car-passenger and taxi trip matrices. For consistency with the logsum calculations, these calculations were made prior to the pivoting process but on the final iteration of the convergence process. Table 3 shows the results of the calculation.

Table 3:Average Week Day Consumer Benefits of the ATM scheme by period using ROH method on road user commute matrices
(per average day)

	Time Period				
	AM	IP	PM	OP	Total
Benefits in £'s	2,798	121	8,010	49	10,979

Most of the benefits are accruing during the AM and PM periods as the scheme provides additional capacity during these periods only. There are some secondary impacts in the IP and OP periods due to the time-of-day shift of car drivers that the model predicts from these periods into the peak periods.

Comparing the results with the logsum calculations, we see that the headline benefit is 9% higher with ROH than with logsums. One aspect of the difference would be that driving costs have not been included in the ROH calculations and these would be expected to increase, as travel by motorways would typically involve longer routes than on lower-level roads. Further differences will come from the linearity approximation of the ROH and from more detailed aspects of the processes.

The ROH method gives a useful (if extreme) insight into the distribution of benefits by time period, but is not able to emulate the segment detail provided by the logsum method.

4.2 Road user charging scenario

In contrast to the simple scenario of highway improvement for a limited number of roads, the second test made is for a comprehensive package of improvements to public transport, highway capacity and the central policy of road user charging.

The charging scheme is based on a number of cordons separating the component parts of the conurbation, with charges up to £2.50 for a single crossing of cordons but no charge for movements entirely within a cordon.

The impacts of the scheme are very limited in terms of the numbers of trips made on each mode, except for a large increase in Metro travel, corresponding to major improvements of that network. However, because of the nature of the charging scheme, cordon crossings diminish by up to 20%, while shorter-distance car traffic is predicted to increase substantially.

The benefits assessed by the logsum approach are shown in Table 4.

Table 4: Logsum-based benefits of charging scheme
(per average day)

Benefits in £'s		**Car Availability Segments**						
Cost coefficient (0.508)		**1**	**2**	**3**	**4**	**5**	**6**	**Total**
Pass Ownership Segments	**1**	328	456	-137	124	-258	39	**554**
	2	191	430	-431	-526	-1,100	-342	**-1,778**
	3	57	-105	-212	-305	-681	-173	**-1,418**
	4	20	34	-41	-31	-102	-17	**-137**
	5	4,009	7,588	-27,232	-22,450	-68,586	-11,963	**-118,635**
	Total	**4,605**	**8,404**	**-28,052**	**-23,188**	**-70,728**	**-12,455**	**-121,415**

Here we see that the benefit to commuters of this scheme is substantially negative. However, against this must be set the revenues of the scheme, which would be several times higher, but against which costs of operation and enforcement must be set. Depending on the use made of that money, and on the costs of the other improvements included in this package, the net benefit could be positive or negative. The result could also be different if the income-segmented version of the model could be used.

We also see that almost all the benefit and disbenefit impacts those without PT passes. Within that group, those with little access to cars benefit from the public transport improvements, while car users have substantial disbenefit and heavy car users the highest levels of disbenefit.

5. ADVANTAGES AND DISADVANTAGES OF THE LOGSUM METHOD

Advantages and disadvantages can be considered as theoretical and practical.

5.1 Theoretical advantages

Accuracy: Logsums are based on actual demand curves while the ROH method assumes linearity of the demand curve.

Consistency: Logsum calculations use consistent coefficients for different attributes as in the demand response model, hence reflect appropriate valuation of the attributes as in the model.

New alternatives: Using the logsum method to calculate consumer surplus change is relatively straightforward as compared to the ROH method which involves several approximations and is quite complicated (Nellthorp and Hyman, 2001).

Soft measures: There is often a need to assess the impact of measures that are not included in standard appraisal procedures. The benefits of such measures can be evaluated using the logsum method if the attributes related to these improvements can be incorporated in the utility definition with appropriate coefficients obtained (e.g.) from Stated Preference analysis methods.

5.2 Theoretical disadvantages

Cardinal Utility: As mentioned earlier (see Section 2) logsum calculations (arguably) retain cardinal utility elements and hence are not consistent with classical welfare economics.

Non-linear formulation: Models that feature a non-uniform cost formulation in the utility function pose problems for conversion to money units. The non-linear formulation of PRISM (for example) is a practical application issue but carries theoretical problems which are resolved by making assumptions.

Income effects: If discrete choice models include income effects in the utility specification then the logsum method of calculating surplus needs to be replaced. However, the ROH calculation would not solve these problems either.

Constrained valuation: The use of the logsum approach means that an appraisal is conducted using the utility coefficients of the demand model, which enforces consistency within an application but which may prevent consistency between different parts of the country.

5.3 Practical advantages

Very easy: As the logsums are already calculated as part of the demand response model, the benefit can be quite easily calculated without any extra effort.

Detailed outputs: Logsums provide detailed information about the distributional consequences of a policy or scheme using the inherent segmentation of the model which otherwise would take significant effort to derive externally using ROH.

Further analysis: Logsums can further be used in accessibility modelling, land-use/ transport interaction modelling etc. to aid more detailed analysis of the impacts of policies and interventions.

5.4 Practical disadvantages

Zero Cost alternatives: Treatment of zero cost alternatives has to be based on the assumption that the marginal utility of cost for users of zero-cost alternatives is the same as that for the rest of the users.

Benefits: Allocation of benefits by choice group (e.g. mode or time period), which is difficult in a logsum application, tends to be easier for understanding the impacts of the scheme than allocation by segment.

Pivoting: Logsums calculated from models which are subsequently pivoted on a base matrix are inconsistent with the matrices used in assignment.

6. CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The use of the logsum method in standard GEV models to calculate consumer surplus is well established in the RUM theory, although there is an ongoing debate on the cardinal nature of RUM applications which includes the logsum measure. Calculation of consumer surplus using the logsum measure, in spite of being relatively straightforward, is rare.

The results obtained for consumer surplus change of commuters due to a highway network intervention; and a RUC policy package illustrate the strengths of the logsum method. Segmentation analysis of benefits, enabled by the inherent segmentation of the demand model, when using the logsum method is seen as the most useful tool to assess the distributional impacts of the scheme and the policy tested. High car-availability segments of the population are observed to benefit the most in case of the highway network improvement and disbenefit the most in case of implementation of the RUC policy package. Low car-availability segments of the population gain some benefits due to the PT improvements that are implemented as part of the RUC policy package. Comparison the logsum based results with the results of the ROH method exhibit reasonable consistency between the two methods, given also that the comparison could not be completed in every detail.

A list of the advantages and disadvantages of the logsum approach, in comparison to the ROH, have been made. This comparison is not decisive either way at present, but it is possible that several of the problems found with the logsum approach could be solved or alleviated by further research.

Apart from the issues of cardinality assumptions and non-linear cost formulation the logsum method proves to be theoretically accurate and practically useful.

6.2 Recommendations

The logsum approach appears reliable and convenient and can be used in circumstances where it has particular advantages over the ROH approach, such as in cases with new alternatives or ‘soft measures’.

Model segmentation could be designed to include segmentation that provides more useful information using logsum based consumer surplus analysis such as income segmentation. In particular income segmentation would be most useful for assessing distributional effects and allowing a re-balancing of the appraisal to meet the needs of decision-makers.

Use of the ROH method as a complementary analysis tool along with the logsum method is recommended for some cases as it would ensure consistency and add to the scheme impact analysis. Mode, time-period etc. distribution of benefits can be assessed using the ROH method for supporting the conventional appraisal of schemes and policies and the more detailed distributional impact analysis can be carried out using the logsum method.

The theoretical debate regarding the cardinality assumptions underpinning logsums has to be concluded with recommendations on methods to reconcile RUM applications to classical welfare economics.

With the new version of PRISM including income as one of the dimensions of segmentation, a similar analysis of the schemes tested here to evaluate their impact of different income groups would be useful.

Use of logsums in further applications to evaluate accessibility of different parts of the network with and without a scheme as a spatial analysis tool would be beneficial for several studies and to calculate social exclusion indicators etc.

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