DESTINATION SAMPLING IN FORECASTING: APPLICATION IN THE PRISM MODEL FOR THE UK WEST MIDLANDS REGION

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1. INTRODUCTION

Model run time is an important consideration in strategic transport modelling, because it determines the speed with which model runs can be carried out and the cost of those runs. Run times are also seen as a key measure of the flexibility of strategic models as they determine the number of scenarios that can be evaluated for a scheme appraisal, or the number of policy options that can be compared in a finite amount of time.

At the leading edge of disaggregate modelling, model runs can take several days even on modern computers and run time increases with model complexity. The greater the range of policy options a model is required to embrace, the greater the level of complexity that is required.

The PRISM model is a highly complex strategic transport model for the West Midlands region, developed by RAND Europe and Mott MacDonald for the Highways Agency of the UK, the seven local authorities of West Midlands county and CENTRO, the regional public transport agency (van Vuren *et al*, 2004, also see www.prism-wm.com). In a normal application of PRISM the demand response models and assignment models are iterated with a feedback loop amongst them several times to achieve overall convergence. The run time of the PRISM model is a critical issue, especially since it has been extended to include an income segmentation module in order to allow charging policies to be assessed in more detail. The addition of the income segmentation module has increased run times by about a factor of two to three.

Destination sampling is a method by which the run time of a disaggregate demand model can be significantly reduced with a minimal loss of accuracy. This is achieved by only performing utility calculations for a sample of destinations, for each origin. Destination sampling in forecasting should be distinguished from alternative sampling in estimation using the method of McFadden (1978). Destination sampling has been used in forecasting in other model systems, such as early versions of the Netherlands National Model and the Stockholm model SIMS (Algers *et al*, 1995), but the present application is believed to be the most fully developed.

For a given origin zone (in a tour based model), the majority of demand will be concentrated in a small number of nearby, large and highly accessible/attractive destination zones. Destination sampling involves selecting a limited sample of destination zones for each origin, taking advantage of this concentration of demand, and carrying out detailed demand calculations for only those destinations. Demand for the remaining destinations is forecast approximately, as a function of demand for the sampled destinations. Using destination sampling, a large proportion of the total demand can be forecast accurately using a fraction of the total number of destinations.

In this paper we describe the implementation of destination sampling in the PRISM demand model and present results to demonstrate the run time savings that can be achieved and the amount of error in the model forecasts when compared to a model run with no destination sampling.

We also highlight the assumptions implicit in the destination sampling calculations, and indicate how these assumptions relate to the amount of error in the forecasts.

2. THE CASE FOR DESTINATION SAMPLING

The PRISM demand model predicts demand for six major travel purposes, each of which involves a loop over 830 tour origins, eight main modes, up to 180 population segments and 898 tour destinations. Before destination sampling was implemented, the run time of the demand model constituted more than half of the total run time of the PRISM model. The remaining run time was consumed by the network assignment component. Less detailed models are used to make predictions for three other travel purposes and for travel to and from the international airport; these models are not considered in this paper – they consume a much smaller fraction of the run time.

For each purpose, the run time of the demand model is approximately proportional to the number of destinations for which utility calculations are performed. As noted in the introduction, for each tour origin the majority of demand is concentrated over a small number of destinations. By performing the utility calculations for only a small sample of high-demand destinations for each origin zone, the run time of the demand model can be reduced significantly. Because the majority of demand is concentrated over these destinations, using an approximation to forecast demand to other destinations should introduce only a small amount of error[1] into the forecasts. A straightforward approach is to use a fixed number of destinations per origin, for a given purpose. The number may differ for different purposes.

Table 1 illustrates the extent to which demand from each origin is concentrated over a small number of destinations in PRISM, for each of the six major travel purposes. For the PRISM base year synthetic[2] demand matrices, it shows the number of destinations that would need to be sampled, per origin, in order to capture a given percentage of overall demand. These figures are based on synthetic model results from a PRISM 2.0 run with full income segmentation, using one P&R station alternative for each destination.

Table 1: Number of destinations to capture a given percentage of demand, by purpose

Percentage of demand	Commute	Shopping	Other	Education		
				Primary	Secondary	Tertiary
70%	55	16	35	5	8	13
75%	73	21	49	7	10	18
80%	99	30	70	11	14	27
85%	137	43	103	16	20	41
90%	197	65	158	27	30	67
95%	310	115	268	54	51	128

Table 2 shows the figures in Table 1 as a percentage of the full set of 898 destinations and, since the run time is approximately proportional to the number of destinations used, suggests the run time savings that could potentially be achieved by this method. Those run time savings are not quite achieved in practice, because there is some run time overhead that is not proportional to the number of destinations, and there is an additional run time overhead associated with destination sampling.

Table 2: Percentage of destinations to capture a given percentage of demand, by purpose

Percentage of demand	Commute	Shopping	Other	Education		
				Primary	Secondary	Tertiary
70%	6.1%	1.8%	3.9%	0.6%	0.9%	1.4%
75%	8.1%	2.3%	5.5%	0.8%	1.1%	2.0%
80%	11.0%	3.3%	7.8%	1.2%	1.6%	3.0%
85%	15.3%	4.8%	11.5%	1.8%	2.2%	4.6%
90%	21.9%	7.2%	17.6%	3.0%	3.3%	7.5%
95%	34.5%	12.8%	29.8%	6.0%	5.7%	14.3%

The extent to which demand is concentrated over a small number of destinations differs for each purpose. The concentration is greatest for primary and secondary education, where 95% of demand is concentrated over the fifty or so most popular destinations for each origin. The concentration is least for commute trips, where more than 300 destinations for each origin are needed to capture 95% of demand. This is to be expected: commuters are known to travel longer distances than primary and secondary school pupils to their place of work/education[3].

3. DESTINATION SAMPLING METHODOLOGY

There are essentially two separate questions to be answered in defining a destination sampling scheme:

1. Which destinations to sample for each origin (for each purpose);
2. How to handle unsampled destinations, which raises two further, closely related questions:
    - How to forecast demand approximately for unsampled destinations;
    - How to forecast tour frequency based on incomplete accessibility information[4].

This section describes the approaches taken to these two issues in PRISM, and outlines some of the alternatives that could be considered.

3.1 Selecting a sample

The results in section 2 show that the level of concentration of demand across the most attractive destinations is different for each purpose. We would also expect *different* destinations to be most attractive for each purpose. Therefore, we use different destination samples for each purpose, of different sizes.

The simplest way to select a sample of destinations for each origin would be to choose the *n* most attractive destinations, based on demand in the base year (and potentially correcting for any changes in attraction variables). This approach would ensure that the largest possible fraction of demand is forecast directly, for a given size of sample *n*.

Unfortunately, choosing only the most attractive destinations would mean that there was a bias in the destination sample towards nearby destinations. Short journeys would be forecast directly, and longer journeys will be forecast approximately by extrapolating the results for short journeys. This could mean that the effect of overall changes in accessibility (caused by e.g. road user charging, or increased congestion) would be underestimated, because the effect on the utility of short journeys would be smaller than for longer journeys. Ultimately, this would be likely to lead to a bias in trip lengths.

It is important that some distant, and therefore unattractive, destinations are sampled to reduce the problem of bias towards short journeys. For PRISM we considered two approaches to this problem: importance sampling, and sampling using groups of zones. Importance sampling was chosen, because it provides a trade off between obtaining some coverage of distant destinations - hence reducing or eliminating bias to trip lengths - and predicting the highest possible fraction of demand directly through sampled destinations.

3.1.1 Importance sampling

In importance sampling, a sample of destinations is selected at random by choosing one destination in turn from the set of all destinations, without replacement. At each step, the probability of each remaining destination being selected is proportional to the expected demand for that destination: the 'expected demand' is a basic estimate of demand in the forecasting year, e.g. calculated by multiplying base year demand[5] by any known changes in attraction variables. If one sample is being prepared for use in all applications, as is the case in PRISM, then changes to the attraction variables are not be known in advance so importance sampling is based purely on demand in the base year.

Importance sampling will ensure that a number of distant, less attractive destinations are sampled for each origin. For commute travel, 95% of base year demand can be accounted for by the 310 most attractive destinations; the remaining 5% being distributed over 588 other destinations. In a sample of 100 destinations created by importance sampling, one might expect five of these 588 other destinations to be selected. In other words, the destinations making up the last 5% of demand might be expected to make up 5% of any sample[6].

While importance sampling captures some distant destinations, to ensure that trip lengths are not biased, the trade off is that it does not achieve the same efficiency - in terms of the percentage of demand captured by a destination sample of given size - as simply choosing the most attractive destinations.

Table 3 shows the percentage of destinations that need to be sampled for each origin, in order to capture a given percentage of overall demand using importance sampling, for each of the major travel purposes in PRISM. These figures are based on synthetic model results from a PRISM 2.0 run with full income segmentation, using one P&R station alternative for each destination.

Table 3: Percentage of destinations to capture given percentage of demand by importance sampling

Percentage of demand	Commute	Shopping	Other	Education		
				Primary	Secondary	Tertiary
70%	11.0%	3.3%	7.7%	1.1%	1.6%	2.9%
75%	13.9%	4.2%	10.1%	1.6%	2.0%	3.9%
80%	17.7%	5.7%	13.4%	2.1%	2.6%	5.3%
85%	22.9%	7.7%	18.3%	3.1%	3.5%	7.7%
90%	30.6%	10.9%	25.7%	4.8%	4.9%	11.6%
95%	43.9%	17.8%	39.1%	8.7%	7.7%	20.3%

These figures can be compared with those in Table 2, which was based on selecting the most attractive destinations for each origin directly.

The random aspect of importance sampling means that the demand captured will differ slightly from one sampling run to another, but the results are very stable. This is demonstrated by Table 4, which shows the corresponding *number* of destinations that need to be sampled for each origin for three importance sampling runs. Of course, the actual zones selected vary from run to run.

Table 4: Number of destinations to capture given percentage of demand: three importance sampling runs

Percentage of demand	Commute	Shopping	Other	Education		
				Primary	Secondary	Tertiary
70%	99 99 99	30 30 29	69 68 68	10 10 10	14 14 14	26 26 25
75%	125 125 125	38 38 38	91 90 90	14 14 13	18 17 18	35 35 35
80%	159 159 159	50 51 51	120 120 120	19 19 19	23 23 23	48 48 48
85%	206 206 206	68 69 68	163 163 164	27 28 27	31 31 31	69 68 69
90%	275 275 275	98 98 98	230 230 231	43 43 43	44 43 44	104 104 104
95%	394 394 394	160 160 160	351 351 351	78 78 78	69 69 69	181 182 181

3.1.2 Groups of zones

Another way to ensure some coverage of distant zones in destination sampling is to define groups of geographically close zones. This approach has previously been used in the Netherlands National Model taking advantage of the zone/sub zone structure of that particular model. In later versions, it was replaced with forecasting based on 100% of destinations.

A destination sample is created (e.g.) by including all zones within a certain distance of the origin, and then sampling one zone from each group beyond this threshold. By cycling through the zones within a group for successive origins, we can ensure even coverage. A second threshold could also be specified beyond which sampling is more sparse.

A refinement to this method might sample all zones up to a certain demand threshold, instead of a distance threshold, after which a zone would be sampled from each group if it is not already represented.

There are drawbacks to the groups of zones method, when compared with importance sampling. It is not as efficient in terms of capturing the maximum percentage of demand for a given number of destinations, because of the need to sample a destination from every group regardless of expected demand. It also imposes the need to define thresholds for full/partial sampling, which may need to be different for each purpose. Finally, forecasting may not be as accurate as it could be, because the zone selected to represent the group may not be the most representative (e.g. closest) sampled zone to each member of the group.

3.2 Forecasting with a sample

Given a sample of destinations for each origin, for each purpose, produced by whatever method, we must find a way to approximately forecast the demand for unsampled destinations. Firstly, the accessibility logsum over all destinations must be approximated for each mode in order for frequency and (where the model is nested logit) mode choice modelling to take place. Secondly, the synthetic demand matrix must be filled in to include unsampled destinations prior to pivoting. These two approximations are intrinsically linked, as the accessibility logsum is the log of the denominator of choice probabilities, which must be extended in an approximate way over unsampled destinations.

For PRISM, we chose to handle unsampled destinations by matching each with its nearest sampled neighbour, and increasing the “size” of each sampled destination in forecasting in order to attract additional demand, which is then distributed over the unsampled destinations.

The following section describes the method in more detail; then two alternatives are outlined. Throughout, the methods described are applied separately for each purpose, and for each origin.

3.2.1 Forecasting based on a nearby zone (“Nearest neighbour” approach)

We forecast demand for unsampled destinations approximately, by associating each with a nearby sampled destination. We reason that changes in accessibility (i.e. the level of service portion of utility) to nearby destinations will be similar, and for the purposes of approximation we assume that they are in fact the same. We also assume that the distribution of demand over destinations is the same for every population segment.

Each unsampled destination is matched to its closest sampled neighbour. For practical purposes closeness is based on the off peak travel time from one to the other in the base year. The sampled destination is then taken to ‘represent’ the unsampled destination in forecasting. *We assume that changes in level of service between base and future year scenarios are the same for the unsampled destination as they are for the sampled destination that represents it.* However, changes in the attraction variables for unsampled destinations are modelled fully.

For each purpose, the mode-destination choice component of the PRISM demand model is a nested logit model with destination below mode or at the same level; in either case the following formulation can be used:

$$P_{m,d,n} = \frac{\exp V_{m,d,n}}{\sum_D \exp V_{m,D,n}} \frac{\left(\sum_D \exp V_{m,D,n}\right)^{\theta}}{\sum_M \left(\sum_D \exp V_{M,D,n}\right)^{\theta}} \tag{0.1}$$

where $P_{m,d,n}$ is the probability of choosing mode m, destination d for a traveller in population segment n. $V_{m,d,n}$ is the utility of mode m, destination d for a traveller in population segment n, and it includes an additive term $\log S_d$, where S_d is the attraction variable for destination d. Therefore, **Error! Reference source not found.** can be reformulated:

$$P_{m,d,n} = \frac{S_d \exp V'_{m,d,n}}{\sum_D S_D \exp V'_{m,D,n}} \frac{\left(\sum_D S_D \exp V'_{m,D,n}\right)^{\theta}}{\sum_M \left(\sum_D S_D \exp V'_{M,D,n}\right)^{\theta}} \tag{0.2}$$

where $V'_{m,d,n}$ is the utility without the attraction variable. It is a linear combination of mode specific constants and linear in parameters level of service terms, all of which may differ by population segment. θ lies between zero and one: for some purposes it is equal to one, in which case the model is multinomial logit (MNL).

For a given mode, if changes in level of service between base and future year scenarios are the same for two destinations and there is no change in attraction variables, then for each population segment the demand for that mode will be divided between the two destinations in the same proportion in the base and future year scenarios. This is because the ratio of demand

$$\begin{aligned} & P_{m,d(1),n} : P_{m,d(2),n} \\ & = S_{d(1)} \exp V'_{m,d(1),n} : S_{d(2)} \exp V'_{m,d(2),n} \end{aligned} \tag{0.3}$$

will not change.

When demand is summed over population segments, it is no longer true that the total for the mode will be divided between the two destinations in the same proportion in the base and future year scenarios. That is, the ratio of total demand

$$\sum_N F_N P_{m,d(1),N} : \sum_N F_N P_{m,d(2),N} \tag{0.4}$$

may change, if the ratio (0.3) is different for each population segment n. F_n is the total number of tours made by travellers in segment n, for the purpose and origin in question.

When forecasting for unsampled destinations, *we assume that the ratio of demand (for a given mode) between an unsampled destination and the sampled destination which represents it is the same for all population segments*. We assume that it is equal to the ratio of total demand (0.4). In the base year, this ratio can be taken from the synthetic base matrix for the purpose in question.

Based on these assumptions, the forecasting procedure is carried out as follows:

1. For each unsampled destination, we identify the nearest sampled destination. That destination will 'represent' the unsampled destination in forecasting.
2. In forecasting, for each mode[7], we increase the attraction of each sampled destination so that it will capture the demand not only for itself, but for all the unsampled destinations that it represents. The exact calculations involved are described in section 3.3. Briefly, we increase the attraction of the destination in proportion to the desired increase in demand. This is calculated based on the demand for each destination in the base year, and any changes in attraction variables.
3. By increasing the attraction of each sampled destination to represent the missing unsampled destinations, we can carry out tour frequency and mode-destination forecasting consistently, using the logsum over the inflated sampled destinations for frequency and mode choice forecasting.
4. After forecasting, we redistribute the demand forecast for each sampled destination over the unsampled destinations that it has been taken to represent. The demand is distributed in proportion to base year demand, adjusted to take account of attraction variable changes.

This procedure is relatively straightforward to implement. It is necessary to calculate an expanded attraction variable (or an expansion factor) for each sampled destination, for each mode; and a redistribution matrix is needed to redistribute the forecast demand over unsampled destinations.

3.2.2 Forecasting based on multiple nearby zones

The nearest neighbour approach above could be taken a step further by forecasting demand for an unsampled destination by associating it with more than one nearby sampled destination.

Each sampled destination gives rise to an estimate of demand for the unsampled destination, by making the same assumption as previously: that demand is divided between the two destinations in a constant ratio (after accounting for attraction changes). We can then forecast approximate demand for the unsampled destination as an average of the estimates produced by the sampled destinations. This could be a weighted average, possibly assigning more weight to the estimates from nearer sampled destinations.

In essence, the unsampled destination is split into a number of smaller destinations, each of which is matched to a different sampled destination.

To represent the unsampled destination in forecasting, we would increase the attraction of each sampled destination enough to capture the desired fraction of the demand for the unsampled destination. The demand would be distributed across the sampled destinations in whatever proportions would be used in the weighted average approximate demand forecast.

This appeal of this approach is that it allows us to take information from more than one sampled destination to forecast the demand for an unsampled destination. Given two neighbouring unsampled destinations, one of which is slightly closer to sampled destination A, and the other slightly closer to sampled destination B, we will forecast demand for each using information from both A and B, but in different proportions. This provides a 'smoothing' of demand changes between A and B.

The disadvantages of the approach are two-fold:

1. The calculations involved are not straightforward to implement in practice, and may involve a slight increase in run time.
2. We must make arbitrary assumptions about how to weight approximate demand forecasts from different nearby destinations.

For these reasons, this approach was rejected in favour of the nearest-neighbour approach.

3.2.3 Forecasting assuming constant accessibility changes

A simpler way of forecasting demand for unsampled destinations is to assume that, between base and future scenarios, the proportional increase in attractiveness for each unsampled zone is equal to the proportional increase in attractiveness of the total set of sampled zones – after taking into account any changes in attraction variables.

We forecast demand for unsampled destinations by dividing the overall demand for each mode between

1. the set of sampled destinations, taken as a whole, and
2. each unsampled destination

in the same ratio that it was divided in the base scenario, after correcting for changes in attraction variables. There is no distinction between different unsampled destinations.

A consequence of this assumption is that we will forecast tour frequency changes by taking the accessibility 'expsum' over the set of sampled destinations for the mode in question, and multiplying it by the ratio (total demand : demand for sampled destinations) for the mode from the base year scenario[8]. In terms of discrete choice modelling, the 'expsum' is the sum of the exponentiated utilities of the destination alternatives. The accessibility 'logsum' is just the logarithm of the expsum.

An advantage of this approach is that the calculations involved are straightforward, although there is a complication in the case of a new development, where a destination might be unavailable in the base scenario but not in future. However, the approach is limited in the sense that all unsampled destinations are treated equally. There is no consideration of the geographical location of the destinations, and any advantage gained by selecting a destination sample intelligently is lost. For these reasons, the approach was rejected in favour of the nearest-neighbour approach.

3.3 Technical considerations

This section lays out some of the technical details of the destination sampling procedure. It specifies exactly how unsampled destinations are be represented by sampled destinations in forecasting, how destinations that are unavailable in the base or future year scenario are handled, and how the added complication of park and ride modelling in PRISM is addressed.

3.3.1 Representing unsampled destinations

In the nearest neighbour approach, unsampled destinations are be represented by associating each with its nearest sampled neighbour. The attraction of each sampled destination will be increased to capture the expected demand for all the unsampled destinations it represents, and the demand forecast will then be redistributed across those destinations.

Specifically, suppose that (for a given purpose, origin and mode) a sampled destination D_0 is chosen to represent each of the unsampled destinations $D_1,\ldots,D_n$. If the number of (synthetic) tours forecast to each destination in the base year is $T_i\ (i=1,\ldots,n)$, the attraction variables in the base year are S_i^b and the attraction variables in a forecasting year are S_i^f, then for each destination we can calculate

$$R_i = T_i \frac{S_i^f}{S_i^b} \tag{0.5}$$

There will not be a problem with S_i^b being zero here, because any destination that is unavailable in the base year will not be involved in the destination sampling process: if it is available in a forecasting year, it will be added to the destination sample and will represent only itself in forecasting (see section 3.3.2).

In the absence of any information about accessibility/level of service changes, we would expect demand in a future year to be distributed over the destinations in the ratio $R_0 : R_1 : \ldots : R_n$. Therefore, in order to capture the attractiveness of the unsampled destinations $D_1,\ldots,D_n$, we increase the attraction of sampled destination D_0 by the factor $\dfrac{R_0 + R_1 + \ldots + R_n}{R_0}$ in forecasting. The attraction of the sampled destination in forecasting will therefore be

$$\tilde{S}_0^f = \frac{S_0^b}{T_0}\left(R_0 + R_1 + \ldots + R_n\right) \tag{0.6}$$

Note that (0.6) is still valid if the attraction variable of the sampled destination becomes zero in a future year, that is if $S_0^f = 0$.

The result of this is two-fold:

1. We increase the accessibility logsum so that it represents the destination alternatives $D_1,\ldots,D_n$, so mode choice and tour frequency modelling can take place;
2. We predict increased demand for destination D_0, which should be distributed over D_0 and the unsampled destinations $D_1,\ldots,D_n$.

After demand modelling using the destination sample, but before pivoting, we can redistribute the demand for D_0 over D_0 and the unsampled destinations $D_1,\ldots,D_n$ by dividing it in the ratio $R_0 : R_1 : \ldots : R_n$.

3.3.2 Unavailable destinations

In either the base year and/or a forecasting year, for a given purpose, any destination could be unavailable either for all modes (if the attraction variable is zero) or one particular mode (e.g. if there is no network connection for a PT mode). The destination sampling procedure needs to account for this.

If a destination is unavailable for all modes in the base year, it will not be selected as part of the destination sample. If it becomes available in a forecasting year, it will be added to the sample, but will not be eligible to represent any unsampled destinations in forecasting. That is because the calculations involved depend on knowing the ratio of attraction variable to demand in the base year (see equation (0.6) above). It will 'represent' only itself.

If a destination is unavailable for some, but not all modes in the base year, then it will not be eligible to represent unsampled destinations in forecasting for those particular modes. Again, this is because the ratio in (0.6) cannot be calculated for those modes. It can happen, therefore, that some *unsampled* destinations will be represented by different sampled destinations for different modes, because the nearest sampled destination is not available for some modes. When selecting the destination sample, an additional step is needed to make sure that at least one available destination is selected for each mode.

If a destination is available in the base year but becomes unavailable in the future year for some or all modes this does not present a calculation problem. The calculations in section 3.3.1 can still be carried out whether the destination is sampled or not. However, if a sampled destination becomes unavailable for some modes because of network changes, the result of the calculations will be that zero demand is forecast to all the unsampled destinations that the sampled destination represents, for those modes. This could occur if, for example, a public transport service was terminated. In such a scenario, the key assumption of destination sampling (that level of service changes are the same for nearby destinations) might be considered inappropriate, so a model run without destination sampling would be recommended.

3.3.3 Park and ride modelling

The implementation of destination sampling in PRISM is complicated by the park and ride model used for train and metro modes, for some purposes[9]. The park and ride modelling methodology is described by Fox (2005). Briefly, for train and metro tours, for each origin and destination, a choice between three access modes is modelled:

1. Car driver access, with a choice of park and ride access station;
2. Car passenger access, with a choice of park and ride access station;
3. 'Other' access, meaning the PT network is used for the entire journey.

In addition, it is possible to run the model without park and ride modelling for applications where accurate representation of PT movements is not required. This saves demand model run time. It is equivalent to making car driver and car passenger access unavailable, so that the PT network is used for the entire journey for any train or metro tour.

Park and modelling complicates the destination sampling process. For different destinations, demand will tend to be divided between access modes in different proportions. In particular, an unsampled destination will tend to have demand divided over access modes in a different ratio to the sampled destination that represents it in forecasting. In extreme cases, some access modes may be unavailable for one destination but available for the other.

To overcome this problem, each main mode/access mode combinations is treated as a separate mode in destination sampling, in the same way that main modes are treated separately. That is, the destination sample for each origin must contain at least one destination that is available by each mode, unsampled destinations may be represented by different sampled destinations for each mode where there are availability issues, and the calculations in section 3.3.2 are carried out separately for each mode.

In PRISM 2.0 access mode choice is on the same level as destination choice in the nested logit model structure for all three purposes that use P&R modelling. Therefore, the approach of treating each main mode/access mode combination as a separate mode is consistent with the overall model structure and no further correction is necessary. However, it is possible within the park and ride modelling methodology that access mode choice *could* be nested below destination for some purposes. In such a case, it would be necessary to correct the overall utility of each sampled destination by train or metro with an additional term, because the expansion for each access mode does not generate the correct overall increase in utility.

A destination sample must be created from a base year model run with park and ride modelling switched on, in order to be used for a forecasting run with park and ride modelling switched on. Such a sample can then also be used for a forecasting run with park and ride modelling switched off: the calculations are carried out for 'other' access only.

For an unsampled destination, car driver and car passenger access tours will be distributed over access stations in forecasting in the same ratio that they are distributed for the sampled destination that represents the unsampled destination. This approximation is adequate for most applications: for a detailed study of park and ride access station choice it might not be appropriate to use destination sampling.

4 BASE YEAR PERFORMANCE

This section reports the performance of destination sampling in the base year, in terms of the accuracy of the forecasts when compared to a model run without destination sampling. Base year results can be used to assess the amount of error introduced into the model forecasts by one of the two assumptions of destination sampling: *that the ratio of demand (for a given mode) between an unsampled destination and the sampled destination which represents it is the same for all population segments*. This might be termed the segmentation error.

To assess the amount of error introduced by the second assumption of destination sampling - *that changes in level of service between base and future year scenarios are the same for the unsampled destination as they are for the sampled destination that represents it* - results from future year model runs are required. Results to show future year performance are not yet available.

For commute, shopping and other travel purposes, destination samples and distribution factors have been generated from a base year run with linear income averaging over income segments and with three park and ride station alternatives. The case with three park and ride stations has been chosen because it allows the attractiveness of each destination by train and metro with park and ride access to be measured most accurately in the base year.

For education purposes, destination samples and distribution factors have been generated from a base year run with full income segmentation (there is no park and ride modelling for these purposes).

The samples have been chosen to directly capture 90% of base year demand, with the remaining 10% of demand being forecast indirectly on the basis of base year demand patterns. Table 5 shows the proportion of base year demand actually captured for each purpose. It also shows the percentage of zones sampled, giving an approximate estimate of the run time required for the sampled runs compared with a full run.

Table 5: Demand coverage by destination samples

	Commute	Shopping	Other	Primary	Secondary	Tertiary
Zones	275	98	231	43	44	104
Percentage of zones	30.6	10.9	25.7	4.8	4.9	11.6
Demand Captured	90.37%	89.72%	90.25%	89.98%	89.81%	89.42%

Some purposes fall slightly short of 90% demand coverage. This can be explained by a slight change to the sampling procedure to ensure that at least one sampled destination is available by train, metro and bus. The numbers of zones to be sampled for each purpose were determined on the assumption that the entire sample would be selected by importance sampling.

Results are shown for a base year model run with three park and ride stations and linear income averaging. For commute, shopping and other purposes this is the scenario which was used to generate the destination samples. For education purposes, there is a difference: the samples were generated based on full income segmentation.

4.1 Overall demand by mode and purpose

Table 6 shows the total demand by mode and purpose forecast by a full model run and a run with destination sampling. The difference between total full run and sampling run demand is no more than 5% for any mode and purpose. The greatest differences in percentage terms are for metro tours for other purposes.

4.2 Demand matching by destination

For each mode and purpose, Table 7 shows the total absolute difference between full run and sampling run demand, analysed on an O-D basis. The absolute difference is expressed as a percentage of total full run demand.

The percentages in Table 7 are necessarily larger than those in Table 6, where only differences in total demand were measured. The greatest differences are still for metro tours for other purposes. The biggest difference is just over 7%.

4.3 Demand matching by time period

For assigned modes, tours are distributed across time periods. All modes except for car driver use fixed time period proportions, while for car driver the time of day choice model is run for all destinations (sampled and unsampled), so the distribution of demand over time periods in a destination sampling run will be absolutely consistent with a full run, for a given O-D pair. This has been verified.

4.4 Tour length distributions

Table 7 shows mean tour lengths by mode and purpose for a full run and a sampling run. Table 8 shows standard deviations. Qualitatively, the results are very similar, and we can conclude that the destination sampling run reproduces full run trip lengths well.

Table 6: Total demand by mode and purpose – number of tours

		Car D	Car P	Train (Car D)	Train (Car P)	Train (Other)	Metro (Car D)	Metro (Car P)	Metro (Other)	Bus	Cycle	Walk	Taxi
Commute	Full run	797440	99416	6080	2372	7847	541	229	1430	135982	14721	86874	5089
	Sampling	797440	99417	6092	2384	7865	541	229	1430	135971	14723	86863	5088
	Error	0.00%	0.00%	0.20%	0.53%	0.23%	-0.09%	-0.13%	0.01%	-0.01%	0.01%	-0.01%	-0.01%
Shopping	Full run	207602	108013	2408	555	4297	549	141	1693	157699	3319	63756	2213
	Sampling	207593	108018	2407	551	4275	548	137	1667	157588	3319	63758	2213
	Error	0.00%	0.00%	-0.05%	-0.86%	-0.52%	-0.19%	-2.56%	-1.52%	-0.07%	0.00%	0.00%	0.00%
Other	Full run	232577	97023	1700	898	3404	6	32	467	66506	3299	54364	6312
	Sampling	232581	97021	1698	890	3318	6	31	445	66475	3299	54363	6312
	Error	0.00%	0.00%	-0.11%	-0.96%	-2.52%	-2.36%	-2.94%	-4.76%	-0.05%	0.00%	0.00%	0.00%
Primary	Full run		103708			1426			321	22750	229	176052	687
	Sampling		103605			1377			316	22487	229	175826	687
	Error		-0.1%			-3.4%			-1.6%	-1.2%	-0.1%	-0.1%	0.0%
Secondary	Full run	2506	38367			13982			1556	87995	2699	98713	1350
	Sampling	2516	38334			13779			1550	87927	2696	98580	1349
	Error	0.4%	-0.1%			-1.5%			-0.4%	-0.1%	-0.1%	-0.1%	-0.1%
Tertiary	Full run	46324	8434			10145			632	8901	669	38825	268
	Sampling	46324	8414			9895			610	8846	667	38765	267
	Error	0.0%	-0.2%			-2.5%			-3.5%	-0.6%	-0.3%	-0.2%	-0.3%

Table 7: Absolute difference between full run and sampling run demand

	Car D	Car P	Train (Car D)	Train (Car P)	Train (Other)	Metro (Car D)	Metro (Car P)	Metro (Other)	Bus	Cycle	Walk	Taxi
Commute	0.04%	0.02%	1.98%	4.94%	0.30%	3.55%	6.28%	0.09%	0.02%	0.02%	0.02%	0.05%
Shopping	0.12%	0.01%	1.63%	3.14%	0.71%	4.74%	6.26%	1.77%	0.08%	0.03%	0.00%	0.08%
Other	0.02%	0.00%	2.33%	4.05%	2.94%	4.95%	7.14%	5.24%	0.07%	0.02%	0.00%	0.03%
Primary		0.11%			3.74%			3.42%	1.24%	0.21%	0.14%	0.94%
Secondary	0.45%	0.20%			2.10%			2.24%	0.27%	0.15%	0.16%	0.34%
Tertiary	0.20%	0.36%			2.90%			4.17%	0.68%	0.37%	0.18%	0.59%

Table 8: Mean trip lengths by mode and purpose / km

		Car D	Car P	Train (Car D)	Train (Car P)	Train (Other)	Metro (Car D)	Metro (Car P)	Metro (Other)	Bus	Cycle	Walk	Taxi
Commute	Full run	28.4	37.1	49.9	41.0	55.0	19.6	17.8	32.2	15.6	10.1	3.0	16.1
	Sampling	28.4	37.1	49.8	41.0	55.0	19.5	17.8	32.2	15.6	10.1	3.0	16.1
Shopping	Full run	13.7	18.0	30.5	30.9	32.0	12.8	11.9	31.2	11.0	5.8	3.4	10.1
	Sampling	13.7	18.0	30.6	31.0	32.1	12.8	11.9	31.4	11.0	5.8	3.4	10.1
Other	Full run	10.8	36.0	37.9	36.7	44.1	18.0	17.6	43.1	14.3	10.3	2.6	12.3
	Sampling	10.8	36.0	37.9	36.7	44.3	17.5	17.5	43.5	14.3	10.3	2.6	12.3
Primary	Full run		7.7			29.6			31.9	13.3	3.9	1.6	6.0
	Sampling		7.7			29.7			32.1	13.3	3.9	1.6	6.1
Secondary	Full run	15.7	10.2			23.0			26.0	12.7	5.9	2.5	12.6
	Sampling	15.7	10.2			22.9			25.9	12.7	5.9	2.5	12.6
Tertiary	Full run	10.6	21.9			48.7			38.2	16.0	7.3	1.9	10.4
	Sampling	10.6	21.9			48.9			38.3	16.0	7.3	1.9	10.3

Table 9: Standard deviation of trip lengths by mode and purpose / km

		Car D	Car P	Train (Car D)	Train (Car P)	Train (Other)	Metro (Car D)	Metro (Car P)	Metro (Other)	Bus	Cycle	Walk	Taxi
Commute	Full run	33.1	38.3	53.8	41.2	56.1	14.1	14.0	19.6	11.9	9.0	3.8	17.4
	Sampling	33.1	38.3	53.8	41.1	56.1	14.0	13.9	19.6	11.9	9.0	3.8	17.4
Shopping	Full run	15.7	19.1	24.9	27.4	24.1	10.4	10.2	20.7	8.9	6.4	3.9	10.2
	Sampling	15.7	19.1	24.9	27.5	24.1	10.3	10.1	20.8	8.9	6.4	3.9	10.2
Other	Full run	18.6	35.2	41.3	41.5	42.9	13.6	13.8	28.0	12.3	9.8	3.6	13.2
	Sampling	18.6	35.2	41.3	41.5	43.1	13.1	13.6	28.3	12.3	9.8	3.6	13.2
Primary	Full run		9.3			21.0			20.6	10.4	2.7	2.5	4.4
	Sampling		9.3			21.2			20.6	10.3	2.7	2.5	4.5
Secondary	Full run	13.0	9.4			16.3			17.4	10.0	5.4	2.9	9.8
	Sampling	13.0	9.4			16.2			17.4	9.9	5.4	2.9	9.8
Tertiary	Full run	12.9	22.7			40.0			26.2	13.3	5.0	2.6	8.1
	Sampling	12.9	22.7			40.3			26.3	13.3	4.9	2.6	8.0

5 RUN TIME SAVINGS

Table 10 shows approximate model run times from an iteration of the PRISM demand and assignment models in forecasting. The figures are taken from a model run with full income segmentation, and one park and ride station alternative for each destination. The numbers of zones in Table 5 have been used, so that 90% of demand is forecast directly through the destination sample for each purpose.

Table 10: Approximate run times for one iteration, with and without destination sampling (hours)

	Demand Model				Demand Model Total	Assignment Model	Total	Demand Model / Total
	Home-based Work, Shopping & 'Other'	Home-based Education	Other Models	Pre- and Post-Processing				
No sampling	12	2.5	0.5	3	**18**	6	**24**	75%
Sampling	4	1	0.5	3	**8.5**	6	**14.5**	59%

The figures show that destination sampling yields a reduction of more than 50% in the run time of the demand model. This translates into a reduction of approximately 40% in the run time of a model iteration.

6 CONCLUSIONS

Destination sampling in forecasting has been implemented in the PRISM model, using the methodology described in this paper. The method takes advantage of structure of the PRISM demand models, with destination choice nested below or at the same level as mode choice in the model tree structure. It results in a significant reduction in model run time (around 40% in this case). Results for the base year show that the segmentation error is small. Although comparative results from forecasting runs with and without destination sampling were not available in time for publication, the model application team have reported no problems with the model forecasts produced by destination sampling.

For large scale modelling applications, destination sampling would appear to provide a valuable saving in model run time without sacrificing the quality of model forecasts.

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[1] 'Error' here is measured relative to the forecasts of a model run without destination sampling, which are taken as 'correct'.

[2] That is, the matrices forecast by the behavioural model in the base year, before "pivoting" against observed base year flows.

[3] The longer distances travelled by secondary pupils (relative to primary) are offset by the fact that secondary schools are larger and therefore occur in fewer potential destination zones.

[4] PRISM tour frequency models incorporate the logsum over mode/destination alternatives, to forecast frequency responses to changes in accessibility.

[5] This could be observed or synthetic demand.

[6] In fact the probability of a given destination being selected for the destination sample is not straightforward to calculate.

[7] The ratio of demand over destinations will be different for each mode, because it depends on level of service differences, which will be mode dependent. Therefore the 'inflation' of sampled destinations and redistribution of demand over unsampled destinations must be carried out separately for each mode.

[8] For consistency with the estimation of the frequency models, this must be the synthetic base scenario.

[9] Namely commute, shopping and 'other' purposes.